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An optimization model of a New Zealand dairy farm

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ABSTRACT

Optimization models are a key tool for the analysis of emerging policies, prices, and technologies within grazing systems. A detailed, nonlinear optimization model of a New Zealand dairy farming system is described. This framework is notable for its inclusion of pasture residual mass, pasture utilization, and intake regulation as key management decisions. Validation of the model shows that the detailed representation of key biophysical relationships in the model provides an enhanced capacity to provide reasonable predictions outside of calibrated scenarios. Moreover, the flexibility of management plans in the model enhances its stability when faced with significant perturbations. In contrast, the inherent rigidity present in a less-detailed linear programming model is shown to limit its capacity to provide reasonable predictions away from the calibrated baseline. A sample application also demonstrates how the model can be used to identify pragmatic strategies to reduce greenhouse gas emissions.

Key words: dairy system, farm modeling, nonlinear optimization

INTRODUCTION

The potential of grazing systems to increase global milk production is high, given the increasing costs associated with high levels of supplementation and the environmental and welfare concerns associated with intensive dairy production (Dillon et al., 2005). Indeed, the total costs of milk production worldwide have been shown to decline nonlinearly as the proportion of grass contained in cow rations increases, with the greatest benefits observed in pasture-based systems, such as those most popular in New Zealand (Dillon et al., 2008). Nevertheless, pasture-based dairy farms are complex systems in which producers must consider multiple interactions between pasture growth and decay, supplement use, individual animal intake and efficiency, and herd size and structure.

Mathematical programming is an optimization technique that has been broadly applied to analyze the integrated management of the multiple components within complex grazing systems (Cartwright et al., 2007). Primary applications include the assessment of agricultural innovations, evaluation of alternative management practices, experimental design, policy analysis, and research prioritization (Pannell, 1996). Berentsen and Giesen (1995) described a linear programming (**LP**) model of a Netherlands dairy system in which pasture growth was defined in terms of an annual total, and pasture quality was fixed. McCall et al. (1999) presented a comprehensive LP model of a dairy farming system in which the length of grazing rotations was optimized. However, pasture residuals, digestibility, and growth were fixed in each period to maintain tractability. Neal et al. (2007) used a detailed LP model to identify the most profitable mix of 36 alternative forage combinations on a farm in New South Wales, Australia. However, the focus on the evaluation of alternative forages meant that forage residuals, digestibility, and growth were fixed in each period. Doole (2010) extended the model of McCall et al. (1999) to incorporate a link between production and nitrate emissions from multiple farms. However, this work retained fixed pasture residuals, digestibility, and growth to maintain tractability and reduce data requirements.

The objective of this paper is to describe a nonlinear programming (**NLP**) model of a New Zealand dairy farm that incorporates several important processes within pasture-based dairy systems that are not considered in previous frameworks. The framework—the integrated dairy enterprise analysis (**IDEA**) model—is the first optimization model of a grazing system to consider (1) postgrazing residual mass as a decision variable of the producer, (2) pasture growth and digestibility that differ with residual pasture mass and rotation length, (3) pasture utilization that varies by stocking rate, and (4) different levels of intake regulation. Model output is demonstrated to match data from system experiments with reasonable accuracy given these extensions. In

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contrast, the rigidity of a less-detailed LP model containing a high number of fixed relationships is shown to restrict its predictive capacity.

Optimization allows the efficient identification of profitable system configurations, which can be time-consuming if manual trial-and-error is used, particularly in complex farming systems. Moreover, shadow prices that indicate the marginal value of different quantities are computed automatically during solution. For example, the IDEA model computes the shadow price of feed energy, the value of an additional megajoule of metabolizable energy, in all fortnights over an average year. Also, the use of constrained optimization allows the natural addition of constraints to identify how a system can optimally respond to new restrictions to farm management, such as constraints on greenhouse gas (**GHG**) emissions. However, optimization models require a certain structure and size to remain tractable. For example, it is problematic to include integer variables in nonlinear optimization models. Additionally, all meaningful mathematical models of grazing systems require a large number of assumptions to be made, given that these systems are inherently complex and dynamic (Kingwell, 2011). A key example is that most models describe farmers as profit maximizers, but in reality producers consider multiple, interacting objectives, such as risk, leisure, and sustainability.

MATERIALS AND METHODS

Model Description

This section describes a nonlinear optimization model of a New Zealand dairy farming system. The model involves a single management year defined from July 1 to June 30. The year is divided into 26 fortnights (14-d periods) to provide insight into the temporal allocation of feed. The first period follows the last in a cyclical fashion to describe decisions that span the last and first periods. The model was constructed to represent farming systems in the Waikato region of New Zealand, the primary dairy farming region in the country.

The model integrates information from a wide range of sources. Many of the coefficients are drawn from the literature and industry publications, especially DairyNZ (2010, 2011). A more detailed description of the model and the sources of model coefficients are provided in Doole et al. (2012). The IDEA model is solved with the CONOPT3 solver in the General Algebraic Modeling System 23.0 (Brooke et al., 2008).

Overview

The solution algorithm for the NLP model identifies the set of decision variables that maximizes operating profit, which is total revenue minus fixed and variable costs. Decision variables in IDEA represent the key management decisions of farmers. These include crop area, type and amount of supplement to import, lactation length, pregrazing and postgrazing pasture biomass, rotation length, silage conservation, and stocking rate. Decision variables are selected by the solution algorithm to maximize operating profit subject to the key constraints facing producers on a typical farm. Key constraints represent cost and quality of supplements, cow energy demands, cow intake, cow reproductive capacity, farm area, and the quantity and quality of pasture available under different management plans.

Feed supply consists of pasture, supplement, and crops. A fixed farm size is allocated between grazing, silage conservation, and cropping in the land use module (gray box in Figure 1) in each period. The pasture module determines the quantity and quality of grazed pasture based on residual pasture mass and rotation length decisions. A feature of IDEA is that livestock intensity and lactation length also drive pasture utilization. The supplement module (gray box in Figure 1) ensures that supplement supply and demand are balanced while accounting for different degrees of supplement utilization. The degree to which potential intake decreases with supplement intake is computed in the substitution rate module. The cow module describes the production, energy demand, and potential intake of each cow type (Figure 1). These differ for each cow type based on age, calving date, degree of intake regulation, genetic merit, and lactation length. Energy supply and energy demand are balanced within the intake constraints of the herd in the integration module, with an overall goal of maximizing operating profit, which is determined in the profit module (Figure 1).

Land Use Module

In any given fortnight, a proportion of the farm can be grazed directly by the cow herd, cut for silage conservation in spring or summer, or cropped (available crops are maize silage and turnips) at the appropriate times of the year. The model determines the optimum area allocated to each of these activities.

The length of a fortnight is $\delta = 14$ d. Time index $i = [1, 2, \ldots, 26]$ denotes the fortnight in which an area of pasture was previously grazed or harvested for silage. In comparison, time index t, where $t = [1, 2, \ldots, 26]$, denotes the fortnight in which an area of pasture is currently grazed, harvested for silage, or rested for future use. An additional index $u = [1, 2, \ldots, 26]$ is used where a (future) activity occurs in a period greater than t.

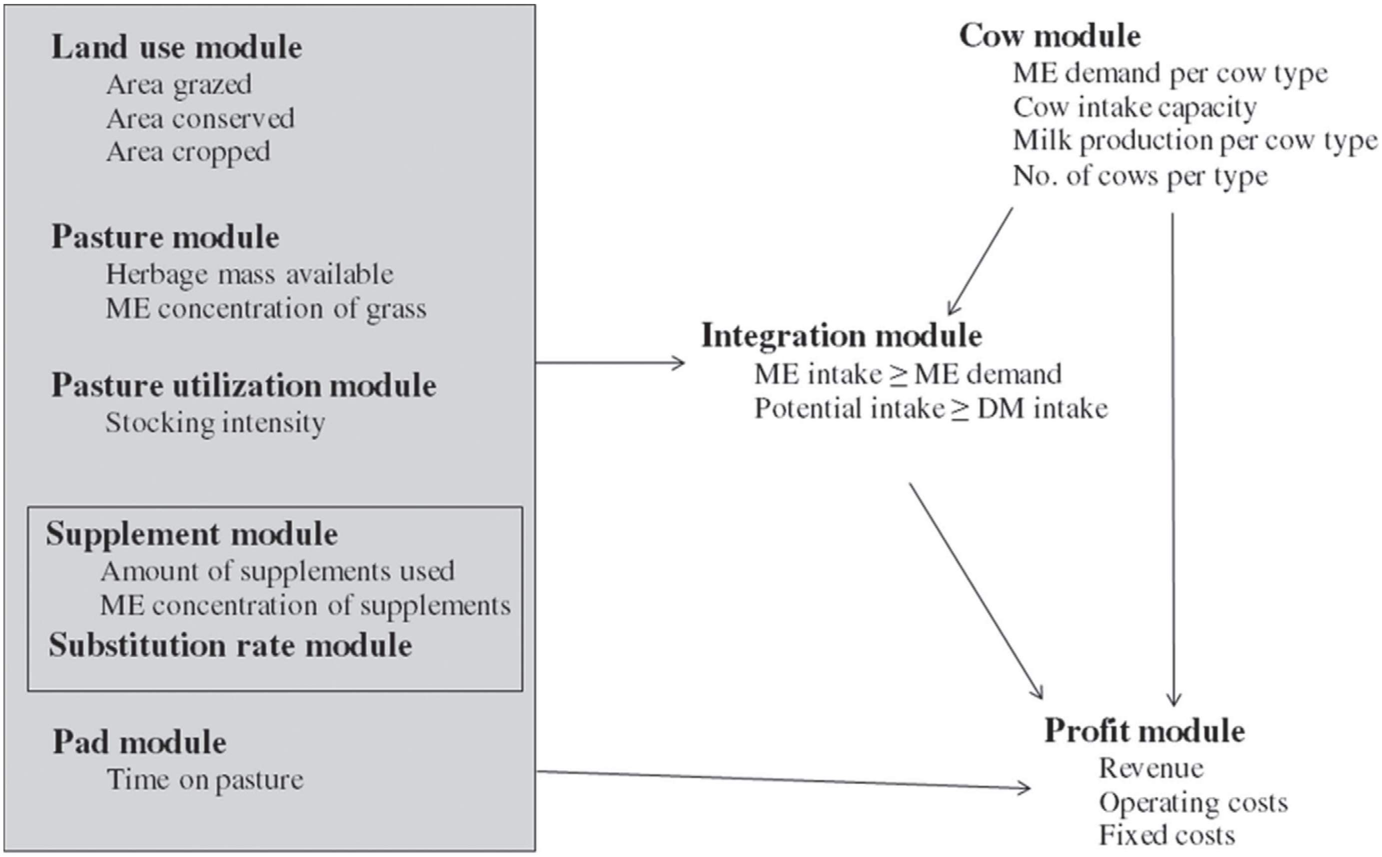

Figure 1. Conceptual diagram of the structure of the integrated dairy enterprise analysis (IDEA) model. The gray box describes the modules that determine energy supply, whereas energy demand is computed in the cow module.

Two residual indices are defined: r_i = the pasture mass (t of DM/ha) that exists after the paddock was grazed or cut for silage in period i; and r_t = the pasture mass (t of DM/ha) that exists after the paddock was grazed or cut for silage in period t. The set of potential residuals is the same for both indices, with $r_i = r_t = \{0.8, 1, \ldots, 2.6\}$ t of DM/ha. However, r_i can be—and typically is—different from r_t for any grazing strategy determined by the optimization algorithm.

Two variables denote the area utilized over fortnight t for grazing or silage production:

- A^G_{i,t,r_i,r_t} = the area (ha) of pasture grazed at time t to a postgrazing residual of r_t that was last grazed or ensiled in period i to a residual of r_i. This decision variable drives pasture eaten from the grazing rotation.
- A^S_{i,t,r_i,r_t} = the area (ha) of pasture ensiled at time t to a postensilement residual of r_t that was grazed or ensiled in period i to a residual of r_i.

Another equation requires residual lengths to remain consistent between periods:

$$\sum_i \sum_{r_i} \left[A^G_{i,t,r_i,r_t} + A^S_{i,t,r_i,r_t}\right] = \sum_u \sum_{r_u} \left[A^G_{t,u,r_t,r_u} + A^S_{t,u,r_t,r_u}\right]. \qquad [1]$$

Decision variables denote the area (ha) allocated to turnips (TU) and maize (MA). The first is a forage crop, whereas maize is harvested for silage. All land is regrassed after crops have been utilized. A proportion of a typical New Zealand dairy farm is also regrassed each year without crop establishment, primarily to replace degraded pasture. The area (ha) on which this is done is denoted RG. The total area that is regrassed each year (AR) after crops or degraded pastures are removed is calculated as follows:

$$AR = TU + MA + RG. \qquad [2]$$

Land use allocation in any period t is

$$
\begin{aligned}
c_{z1} = AR &+ \sum_{i}\sum_{r_i}\sum_{r_t}\left[A^{G}_{i,t,r_i,r_t} + A^{S}_{i,t,r_i,r_t}\right] \\
&+ \sum_{i}\sum_{u}\sum_{r_i}\sum_{r_u}\left[A^{G}_{i,u,r_i,r_u} + A^{S}_{i,u,r_i,r_u}\right]_{\forall i\neq t, t>i, u>t, u>i} \\
&+ \sum_{i}\sum_{u}\sum_{r_i}\sum_{r_u}\left[A^{G}_{i,u,r_i,r_u} + A^{S}_{i,u,r_i,r_u}\right]_{\forall i\neq t, i>t, u>t, i>u} \\
&+ \sum_{i}\sum_{u}\sum_{r_i}\sum_{r_u}\left[A^{G}_{i,u,r_i,r_u} + A^{S}_{i,u,r_i,r_u}\right]_{\forall i\neq t, t>i, t>u, i>u},
\end{aligned}
\qquad [3]
$$

where c_{z1} is total farm size. The first line in Eq. [3] accounts for the area removed from the grazing rotation (AR) and current land use. The second and third lines define land that is being rested for future use. The second line describes cases in which $u > t > i$; for example, where land was last used in period 5, it is currently period 13, and it will be utilized again in period 17 ($u = 17 > t = 13 > i = 5$). The third line describes cases in which $i > u > t$. The last line describes cases in which $t > i > u$. The third and fourth lines define cases where pasture is rested for a period encompassing both the last and first fortnights, consistent with the equilibrium structure of the model.

Pasture can be grazed before crops or new pasture are sown and after new pasture has successfully established following these land uses. The area of pasture available within area AR in period t is:

$$A^{R}_{t} = TU_{t=[23,7]} + MA_{t=[23,7]} + RG_{t=[23,18]}, \qquad [4]$$

where $AR \geq A^{R}_{t}$. Subscripts on the right-hand side of Eq. [4] denote the periods when pasture is available on land allocated to a given crop or regrassing activity. For example, pasture is only available on land on which a turnip crop (TU) is planted over fortnights 23 to 7, as it is in crop the remaining time. Ten percent of the farm must be regrassed each year, in accordance with industry practice.

Pasture Module

The detailed pasture module represents several key processes. For example, the model considers that grazing to a lower postgrazing residual can (1) increase the amount of herbage available in the current grazing, (2) reduce the digestibility of material ingested in the current period due to the grazing of older plant material, (3) reduce subsequent pasture production due to defoliation of photosynthetic material, (4) decrease future losses to senescence, and (5) increase digestibility of regrowth due to reduced shading.

The grazing rotation yields a base amount of pasture eaten (Q^{G}_{t}, t of DM) in each period. The base amount of pasture eaten depends on the postgrazing residuals of the previous and current grazing events (denoted by r_i and r_t, respectively) and the growth that has occurred since the previous grazing.

A simulation model of pasture dynamics (Romera et al., 2009) incorporating daily climate information was extended to incorporate tissue age structure. It was subsequently calibrated to appropriate experimental information to generate pasture growth and digestibility data for the optimization model. This was necessary given the unavailability of appropriate trial data. The model was run individually for each postgrazing residual mass that existed after grazing in period i ($r_i = \{0.8, 1, \ldots, 2.6\}$). The model was used to estimate the age structure and amount of pasture available to the herd in each of the 7 subsequent fortnights (98 d). Growth differs according to the observed climate conditions and the postgrazing residual mass used as the initial condition for the dynamic process. The postgrazing residual mass in the current fortnight ($r_t = \{0.8, 1, \ldots, 2.6\}$) then determined how much of this estimated mass was eaten and what was the age structure of the ingested material. Mean pasture production and quality were determined for the decade 2000–2009 to account for temporal variation in pasture attributes, at least to some degree.

Residual mass decisions were not studied specifically for the grazing of pasture on the cropped and regrassed area AR. Instead, a set of typical residual masses was determined for this area, pasture growth was assumed constant given this set of residuals, and pasture grown was grazed in each period. This approach was necessary to maintain model tractability and follows the method of McCall et al. (1999). The amount of pasture produced on the cropped or regrassed area (Q^{R}_{t}, t of DM) was the product of the constant growth over the period during which this area produced grass to be grazed (see Eq. [4]) (q^{R}_{t}, t of DM/ha) and the size of the cropped or regrassed area AR.

The variable N_i denotes the tonnes of N fertilizer applied in time i. Constraints restricted application to 50, 100, and 400 kg every period, 6 wk, and 1 yr, respectively (McCall et al., 1999). Following McCall et al. (1999), additional pasture growth from N fertilizer application was represented separately from the rotational grazing system to maintain model tractability. The total amount of additional herbage mass available to grazing animals in period t arising from the application of N_i $\left(Q^{N}_{t}\right)$ in period i was determined using the decision tree of Zhang and Tillman (2007).

Total herbage mass (Q^{H}_{t}, t of DM) grazed in time t was thus

$$Q_t^H = Q_t^G + Q_t^R + Q_t^N. \quad [5]$$

The total amount of herbage mass (Q_t^S, t of DM) ensiled in period t was the product of the herbage mass ensiled on each unit area (q_{i,t,r_i,r_t}^S, t of DM/ha) and the total area allocated to silage production (A_{i,t,r_i,r_t}^S, ha). The postensilement residual was determined optimally from the set $r_t = [1.4, 1.6, \ldots, 2.2]$ t of DM/ha.

The energy obtained from each source of grazed pasture was determined through multiplication of the amount of feed consumed (t of DM) and the energy of the consumed herbage (MJ of ME/t of DM). The energy of the material grazed in the standard rotation was the product of the digestibility of this herbage and a parameter that converted this digestibility into energy content. The energy content of pasture on areas that have been cropped or regrassed and of pasture arising from N fertilizer application was the mean of that obtained in the standard grazing rotation.

Pasture Utilization Module

Research suggests that a strong, positive curvilinear relationship exists between cow intake and herbage allowance (Maher et al., 2003). This relationship was included in the model to prevent unrealistic levels of herbage utilization being realized at low stocking rates. The concave intake model of Gregorini et al. (2009), based on the formulation of McCall et al. (1986), was used to represent this relationship in IDEA. Pasture utilization (PU_t) was a concave function defined as

$$PU_{cd,t} = 1 - \exp[-K_{cd,ll,t}\ ISt], \quad [6]$$

where IS_t is the instant stocking intensity (cows/ha per day), the number of cows divided by the area grazed on each day during each grazing period. The area grazed depends on the rotation length, which varies from period to period; $K_{cd,ll,t}$ is a variable describing the shape of the pasture-utilization function and is defined for all calving dates cd, lactation lengths ll, and time periods t (see the Cow Module section for a description of the cd and ll indices). The variable $K_{cd,ll,t}$ was computed as follows:

$$K_{cd,ll,t} = 0.011 \cdot \exp[-0.00346 \cdot DC_{cd,ll,t}], \quad [7]$$

where $DC_{cd,ll,t}$ is the number of days of lactation for a cow with lactation length ll at time t since calving occurred in period cd (days/cow). The parameter values defined in this function were from Gregorini et al. (2009).

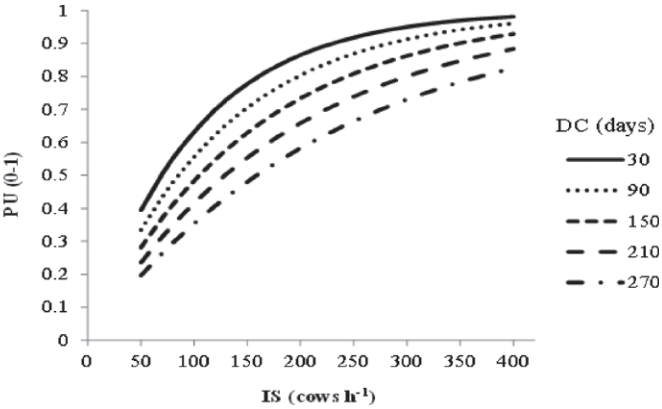

Figure 2. Pasture utilization (PU: proportion of the pasture allocated for grazing that could be eaten) as a function of stocking intensity (IS) and days since last calving (DC).

Equation [6] is illustrated in Figure 2 and implies that pasture utilization increases curvilinearly as stocking intensity increases, consistent with theory (e.g., Maher et al., 2003). It is also evident from Figure 2 that cows possessed a greater drive to utilize more pasture as the number of days since last calving increased.

The function stated in Eq. [6] defines an upper bound on the total amount of pasture eaten. The total pregrazing herbage mass from ground level was defined as PM_t (t of DM). The cow herd can eat only a proportion of the total herbage mass offered to them in each period, determined by PM_t (Eq. [6]). The upper limit for total pasture intake (TA_t, t of DM) in period t was

$$TA_t = \sum_{cd=1}^{9}\sum_{ll=1}^{7} PU_{cd,ll,t} CC_{cd,ll}\left[\frac{PM_t}{TC}\right], \quad [8]$$

where $CC_{cd,ll}$ is the number of cows that calve in cd and have a lactation length of ll, TC is the total number of cows on the farm, and the term in brackets represents total pregrazing herbage mass from ground level per cow (t of DM/cow).

A limit on total pasture intake was then defined as

$$TA_t \geq Q_t^H, \quad [9]$$

where total pasture mass consumed $\left(Q_t^H\right)$ is calculated in Eq. [5].

The relationships between stocking rate, herbage allowance, postgrazing herbage mass, and pasture utilization possessed a high degree of interdependency. For example, higher stocking rates increased pasture utilization per unit area, decreased herbage allowance, and limited total ingestion, which restricted the amount of feed available, which partly determined stocking rate.

The presence of such circular relationships is easily dealt with in mathematical programming, which simultaneously identifies the values of all decision variables in the model during solution.

Supplement Module

Several supplementary feeds are available in IDEA: grass silage made on farm or purchased, maize silage made on farm or purchased, or palm kernel expeller (**PKE**) that is purchased. Turnip crops can also be grazed for forage.

The total amount of grass silage produced on the farm in each period is Q_t^S. The amount of grass silage eaten by cows (t of DM) in each period is F_t^{SG} on the grazing area and F_t^{SP} on a feedpad (a concrete area on which cows are fed for 2 to 3 h/d). The total amount of grass silage eaten is $F_t^S = F_t^{SG} + F_t^{SP}$. The amount of grass silage purchased (sold) is denoted as GAP (GSS). Supply and demand is balanced through

$$\sum_{t=1}^{26} Q_t^S (1 - c_{s1}) + GAP = \frac{\sum_{t=1}^{26} F_t^{SG}}{(1 - c_{s2})} + \frac{\sum_{t=1}^{26} F_t^{SP}}{(1 - c_{s3})} + GSS, \tag{10}$$

where c_{s1}, c_{s2}, and c_{s3} are the proportional loss of grass silage at harvest and storage, feeding on the paddock, and feeding on a feed pad, respectively.

The amount (tonnes) of maize silage purchased (sold) is denoted as MAP (MAS). The amount of maize silage eaten by cows (t of DM) in each period is F_t^{MG} on the grazing area and F_t^{MP} on a feedpad. The total amount of maize silage eaten is $F_t^M = F_t^{MG} + F_t^{MP}$. Supply and demand is balanced through

$$c_{s4}(1 - c_{s5})MA + MAP = \frac{\sum_{t=1}^{26} F_t^{MG}}{(1 - c_{s6})} + \frac{\sum_{t=1}^{26} F_t^{MP}}{(1 - c_{s7})} + MAS, \tag{11}$$

where c_{s4} is maize yield (t of DM/ha) and c_{s5}, c_{s6}, and c_{s7} are the proportional loss of maize silage at harvest and storage, feeding on the paddock, and feeding on a feed pad, respectively.

The amount of PKE purchased is denoted PAP. The amount of PKE eaten by cows (t of DM) in each period is F_t^{PG} on the grazing area, F_t^{PP} on a feedpad, and F_t^{PS} in a dairy-shed feeding system (a system that feeds cows as they are milked in the dairy shed). The total amount of PKE eaten is $F_t^P = F_t^{PG} + F_t^{PP} + F_t^{PS}$. Its use is governed by

$$PAP = \frac{\sum_{t=1}^{26} F_t^{PG}}{(1 - c_{s8})} + \frac{\sum_{t=1}^{26} F_t^{PP}}{(1 - c_{s9})} + \frac{\sum_{t=1}^{26} F_t^{PS}}{(1 - c_{s10})}, \tag{12}$$

where c_{s8}, c_{s9}, and c_{s10} are the proportional loss of PKE at feeding on the paddock, feedpad, and in a dairy-shed feeding system.

The amount of turnips eaten (t of DM) in period t is F_t^T. Supply and demand is balanced by

$$c_{s11} TU = \frac{\sum_{t=15}^{19} F_t^T}{(1 - c_{s12})}, \tag{13}$$

where c_{s11} is the yield of the turnip crop and c_{s12} is the proportional loss of turnips at feeding.

Maximum bounds were set on the consumption of some supplementary feeds. These were set according to industry recommendations reported in DairyNZ (2010).

Substitution Rate Module

Substitution rates vary with pasture intake, as substitution will increase as ingestion limits imposed by potential intake are approached. Linear programming models of dairy systems typically use fixed substitution rates, which reduces their capacity to represent such a relationship (McCall et al., 1999). Thus, the substitution rates used in the model were defined using regression equations estimated from data from 20 trials by Stockdale (2000), which includes the consideration of the level of unsupplemented pasture intake relative to the liveweight of cows. These relationships were suitable for New Zealand application, as substitution rates are relatively universal (Stockdale, 2000).

The substitution rate for grass silage $\left(SR_t^S\right)$ and maize silage $\left(SR_t^M\right)$ is

$$\begin{aligned} SR_t^S = SR_t^M &= -0.26 + 0.17\left[\frac{MI_t}{ML_t}\right] + 0.08 se_t \\ &+ 30\left(\frac{(F_t^S + F_t^M + F_t^P)}{\delta}\right) - 0.04, \end{aligned} \tag{14}$$

where MI_t is mean unsupplemented pasture intake (kg of DM/cow), ML_t is mean cow liveweight (100 kg/cow), δ is the days in a fortnight, and se_t is an index indicating the time of year (+1 for spring, 0 for summer, −1 autumn and winter).

Similarly, the substitution rate for PKE $\left(SR_t^P\right)$ is

$$SR_t^P = -0.26 + 0.17\left[\frac{MI_t}{ML_t}\right] + 0.08se_t + 30\left(\frac{(F_t^S + F_t^M + F_t^P)c_{a5}}{\delta}\right) + 0.04. \quad [15]$$

The substitution rates increased with unsupplemented pasture intake defined per unit of cow liveweight (the term in brackets). For example, if computed on a daily basis, if a 480-kg cow was fed 1 kg DM of maize silage daily in spring, this would offset 0.19, 0.36, 0.54, and 0.72 kg of pasture DM at intakes of 5, 10, 15, and 20 kg of pasture DM for a 480-kg cow. The substitution rates also increased with daily supplement intake (the term in parentheses). For example, if a 480-kg cow was fed 1, 2, 3, or 4 kg DM of maize silage in spring, this would offset 0.54, 0.57, 0.6, and 0.63 kg of pasture DM at a daily pasture intake of 15 kg of pasture DM. Further scenarios are shown in Figure 3.

The substitution rate for turnips is $SR_t^T = 1$ (DairyNZ, 2010).

Cow Module

A total of 15,120 attribute combinations (cow types) describe a large number of different types of cow. Cow types differ according to the following:

- $uf = [1, 2, \ldots, 6]$: These correspond to 0, 10, 20, 30, 40, and 50% decreases, respectively, in the annual energy intake of a fully-fed cow. The energy intake of New Zealand dairy cows is typically reduced below potential for at least a proportion of the year, especially in the period before calving (Macdonald et al., 2008).
- $a = [1, 2, \ldots, 4]$: These represent cows that are 2, 3, 4, and 5+ yr of age, respectively.
- $m = [1, 2, \ldots, 5]$: These represent the genetic merit of cows according to peak milk yield (kg/d).
- $cu = [1, 2]$: These represent standard cows ($cu = 1$) or those to be sold as culls after lactation is complete ($cu = 2$).
- $cd = [1, 2, \ldots, 9]$: These represent calving dates of July 1, July 15, July 29, August 12, August 26, September 9, September 23, October 7, and October 21.
- $ll = [1, 2, \ldots, 7]$: These represent lactation lengths of 180, 210, 250, 265, 280, 295, and 310 d, respectively.

The number of cows with each attribute combination is denoted $C_{uf,a,m,cu,cd,ll}$. The total number of cows (TC) is thus defined

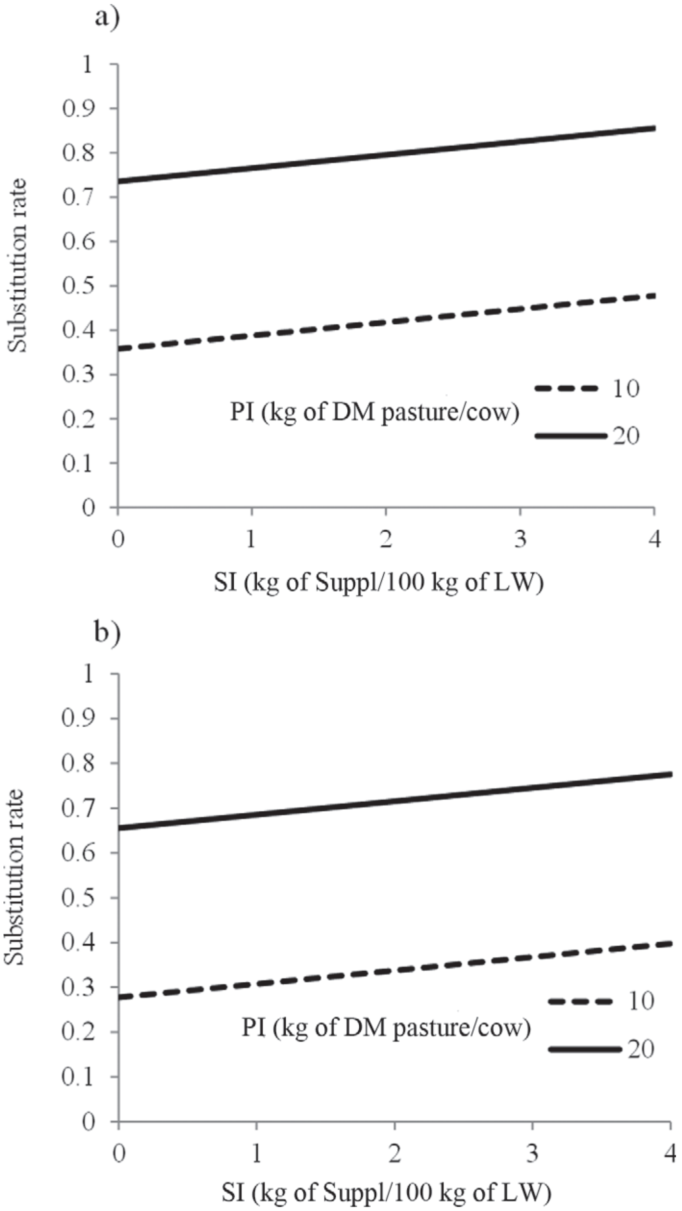

Figure 3. Substitution rate as a function of the supplementary feeding intake (SI) for (a) silages and (b) concentrates. These relationships are computed for 2 levels of unsupplemented pasture intake (PI). LW = liveweight; Suppl = supplement. Data are from Stockdale (2000).

$$TC = \sum_{uf=1}^{6}\sum_{a=1}^{4}\sum_{m=1}^{5}\sum_{cu=1}^{2}\sum_{cd=1}^{9}\sum_{ll=1}^{7} C_{uf,a,m,cu,cd,ll}. \quad [16]$$

The total number of cull cows is

$$TCU = \sum_{uf=1}^{6}\sum_{a=1}^{4}\sum_{m=1}^{5}\sum_{cd=1}^{9}\sum_{ll=1}^{7} C_{uf,a,m,cu=2,cd,ll}. \quad [17]$$

The stocking rate (cows/ha) is the total number of cows divided by the total area of the farm. The number of cows in each age class is determined through

$$\sum_{cu=1}^{2} C_{uf,a+1,m,cu,cd,ll} = C_{uf,a,m,cu=1,cd,ll} SV \quad \forall a = [1,3], \text{[18]}$$

where SV is the survival rate. The survival rate is the sum of the cull rate for disease in adult cows, the natural mortality rate of adult cows, and the empty rate. We assumed that all empty cows were culled from the herd. The empty rate for a given cow type was computed based on the efficiency of reproductive management, cow health, rate of embryonic loss, and the number of heats exhibited during the mating period, which depended on age, changes in cow liveweight before mating, and the conception rate on each heat. These were described through the use of the reproduction management model of Beukes et al. (2010). All cull and dead cows were replaced each year.

The total number of female calves retained was calculated considering the empty rate, the sex ratio of calves, the natural mortality rate of calves, and the proportion of female calves sold. All male calves were sold; thus, all yearlings were female. The total number of yearlings retained was calculated considering the natural mortality rate of yearlings and the proportion of female yearlings sold.

Genetic merit for milk production affects the relationship between cow intake and milk production. Thus, the distribution of genetic merit across the herd in an optimal solution must be consistent with those distributions observed on typical New Zealand dairy farms. Levels of peak milk were derived from data for 850,000 cows from the Livestock Improvement Corporation (Hamilton, New Zealand). The set of all levels of genetic merit to which a cow can belong was described as $mw = [1, 2, \ldots, 5]$. The distribution was defined in each solution through

$$TC \cdot me_{mw} = \sum_{uf=1}^{7} \sum_{a=1}^{4} \sum_{cu=1}^{2} \sum_{cd=1}^{9} \sum_{ll=1}^{7} C_{uf,a,m=mw,cu,cd,ll} \quad \forall mw, \quad \text{[19]}$$

where me_{mw} is the proportion of the cows in a standard herd that possess a given level of genetic merit for milk production mw.

The calving distribution must also approximate those distributions observed on real farms. The user can select a given calving date or the model can select the optimal calving date. The user can also select the proportion of the herd required to calve in each subsequent fortnight. Calving is assumed to take place over an 8-wk period, spread over 5 fortnights, in the baseline situation; 16, 37, 22, 11, and 14% of the herd calved in the first, second, third, fourth, and fifth fortnights, respectively (drawn from unpublished DairyNZ data).

Total milk production was the sum of the annual milk production (t of milk solids, **MS**) of each cow type multiplied by the number of cows of each type.

Integration Module

The level of potential pasture intake, energy demand, and milk production for each cow type was computed outside of IDEA in a detailed optimization model. This model was based on existing simulation models, described by NRC (2001), Johnson et al. (2008), and Freer et al. (2010). Use of an exogenous model to compute these quantities allows thorough consideration of important processes, such as the dynamics of body condition gain and loss, while maintaining the tractability of IDEA.

Potential intake for a given type of cow specifies the maximum amount of feed (in kg of pasture DM) that a cow can ingest in a given period when unrestricted access is given to a feed with a digestibility of at least 80%. (This is measured in terms of pasture DM to ensure that intake substitution is accounted for.) The first integration constraint specified that the total DM intake of the cow herd cannot be higher than the potential intake of the herd in period t. Supplement intake was converted into pasture DM equivalents through the use of substitution rates (Eq. 14–15).

Metabolic energy is expended for gain in body condition, growth, lactation, maintenance, and pregnancy. Metabolic energy was obtained from the ingestion of feed and loss in body condition. The second integration constraint specified that the total energy intake of the cow herd must be higher than the total energy demand of the cow herd in period t. The energy gained through supplement intake (MJ of ME) was calculated through multiplication of the amount of each supplement fed (t of DM) by its average energy content (MJ of ME/t of DM).

Pad Module

A farm can possess a feed pad, a stand-off pad, or both. A stand-off pad involves standing cows on bark chips or other soft surfaces for a proportion of the day to reduce time on pasture. Cows are not fed on a stand-off pad, in contrast to a feed pad. Keeping cows off pasture is a key strategy for reducing soil compaction and reducing nitrate leaching and nitrous oxide emissions by decreasing the amount of urine that is deposited on agricultural soils.

The proportion of the day spent on the feed pad and stand-off pad were decision variables in the optimization. A cow cannot spend more than half of the day on a stand-off pad unless a feed pad is present, or this will lead to detrimentally low levels of feed intake. Putting cows on pads reduces their capacity for pasture consumption, as depicted in Figure 4.

Profit Module

The objective function involved maximization of operating profit (OP_f; \$) for the farm. This is consistent with standard farm-level models (Cartwright et al., 2007; Kingwell, 2011). Solutions were equivalent to those obtained with the maximization of profit per ha (e.g., McCall et al., 1999), as $OP_f = c_{z1}OP_h$, where c_{z1} (farm size) is constant and OP_h is operating profit per ha. Profit per kilogram of MS is also reported in each model solution and can be defined as the model objective.

Operating profit at the farm level was calculated considering income from MS (fat plus protein), cull cows, cull calves, cull yearlings, grass silage that is sold, and maize silage that is sold. Costs included a fixed cost of production defined per hectare, a variable cost of production defined per cow, cost of grass silage conservation, purchase of grass silage and maize silage, purchase of PKE, purchase and application of fertilizers (nitrogen and potassic superphosphate), grazing replacement female calves off-farm (grazing payment and transport), crop establishment, establishing and regrassing after crops or directly from pasture, and establishing and maintaining a feed pad or stand-off pad. The coefficients of the profit function were based on current market values in New Zealand and information from the DairyNZ Economic Survey (DairyNZ, 2011).

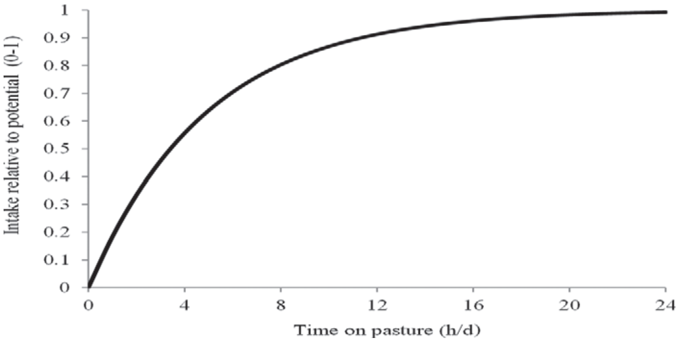

Figure 4. Effect of time on pasture on the intake of grazing cows. Estimated using data from Clark et al. (2010).

RESULTS AND DISCUSSION

Model Runs

In this section, we investigated the capacity of IDEA to predict data from the farmlet trial reported in Macdonald et al. (2008; hereafter referred to as **MD08**). Moreover, output from a less-detailed LP model—based on the framework of McCall et al. (1999)—was generated for comparative purposes. The last section in the Results and Discussion explores the capacity of IDEA to identify cost-effective responses to constraints imposed on GHG emissions (**GHG-e**).

The LP model was constructed specifically for this analysis. It is less detailed than IDEA, incorporating around 7,000 equations and 4,500 decision variables, whereas IDEA has 30,000 constraints and 700,000 decision variables. The LP model incorporates fixed pasture growth, pasture residuals, pasture utilization, potential intake, and substitution rates. Postgrazing mass is fixed according to the grazing strategy used by MD08. For IDEA, the simulation model of pasture dynamics described above was used to estimate pasture growth and digestibility for the location and period of the MD08 study.

The IDEA and LP models were both calibrated to data generated from the 3.1 cows/ha treatment in the MD08 trial. Both models were then used to predict values for the 2.2 and 4.3 cows/ha treatments in the MD08 trial without further calibration. These stocking rates were the lowest and highest, respectively, of those used in MD08 and tested the capacity of both models for meaningful prediction. Stocking rate was fixed in each model, in accordance with the experimental treatments used. Moreover, fixed rates of N fertilizer application were used. However, no further constraint addition was used for calibration, as this restricts the flexibility of the model to respond to perturbations. Following McCall et al. (1999), the objective in the optimization for the validation exercise was to maximize milk production.

Comparison of IDEA and LP with Stocking Rate Treatment of 3.1 cows/ha in MD08

The IDEA model provided a meaningful description of the MD08 treatment incorporating 3.1 cows/ha. The broadest deviations from MD08 data differed little in absolute terms. First, the autumn grazing interval was 14% higher than in the trial data, but corresponded to a difference of only 5 d (Table 1). Second, the level of silage eaten was 8% lower than in the trial data, but corresponded to a difference of only 44 kg of DM/ha. Last, the area of the farm topped was 10% lower than

Table 1. Key farm characteristics recorded for a farmlet stocked at 3.1 cows/ha in Macdonald et al. (2008; MD08) and the equivalent values predicted in the integrated dairy enterprise analysis (IDEA) model and a linear programming (LP) model of the same farming system

	Source of output				
Variable	MD08	IDEA	Diff.[1] (%)	LP	Diff.[1] (%)
Pregrazing mass (kg of DM/ha)	3,355	3,228	−4	3,084	−9
Postgrazing mass (kg of DM/ha)	1,985	1,880	−6	2,038	+3
Grazing interval (d)					
Winter	70	67	−4	72	+2
Spring	30	30	0	34	+12
Summer	25	24	−4	25	0
Autumn	30	35	+14	31	−3
Pasture eaten (kg of DM/ha)	14,322	14,771	+3	15,206	+6
Silage conserved (kg of DM/ha)	806	813	+1	1,172	+45
Silage eaten (kg of DM/ha)	562	518	−8	1,172	+109
Farm topped (%)	65	59	−10	60	−8
OM digestibility (% of DM)	78	78	0	76	−3
Pasture ME (MJ/kg of DM)	11.3	11.25	−1	11	−3
Lactation length (d)	258	260	+1	247	−4
Annual milk production (kg/cow)	4,128	4,105	−1	4,124	−1
Annual milk production (kg/ha)	12,796	12,726	−1	12,784	−1

[1] Percentage difference between the value of the farm characteristic in Macdonald et al. (2008) and the model prediction.

in the trial data, but corresponded to a difference of only 6% of farm area.

The majority of model outputs fit the trial data well (Table 1). Pre- and postgrazing pasture masses were around 5% below the trial data. This stimulated pasture growth—hence pasture eaten was higher by 3% in IDEA—as lower grazing residuals improve light interception and photosynthetic efficiency in perennial-ryegrass swards (Chapman and Lemaire, 1993). The IDEA model provided accurate predictions of OM digestibility and pasture ME, reinforcing the value of representing residual length as a decision variable in a model of a dairy farm (Table 1). The detailed cow model also provided high precision in terms of lactation length and milk production.

The LP framework also provided a meaningful description of the MD08 treatment incorporating 3.1 cows/ha. Output from the LP model fit the trial data for grazing interval as well as the IDEA model did. Moreover, milk production in the LP matched the trial data more closely than IDEA, although lactation length was 11 d (4%) below the MD08 levels. Pasture eaten was 884 kg of DM/ha (6%) above trial data—around twice the magnitude of the deviation reported for IDEA (Table 1)—because pasture growth is constant and there is no relationship between cow intake and herbage allowance. These factors also drove large deviations related to feed conservation. Silage production and feeding were 45 and 109% above trial data, respectively.

Silage conservation and feeding were also the factors that deviated the most from reported outcomes when LP outputs were compared with trial data in McCall et al. (1999). This difficulty in LP models of dairy farms occurs for several reasons. First, use of fixed pasture growth rates fails to account for ryegrass growth dynamics (especially senescence) in paddocks spelled for silage conservation, especially at high levels of pasture mass. Second, use of a fixed residual for silage production in a LP model reduces the flexibility with which silage making can be used in the grazing rotation.

Comparison of IDEA and LP with Stocking Rate Treatment of 2.2 cows/ha in MD08

The IDEA model also provided a very good representation of the low stocking rate (2.2 cows/ha) system in MD08. Pregrazing biomass differed from trial data by 1%, but postgrazing pasture mass was 162 kg of DM/ha (8%) lower. Grazing intervals were also longer in IDEA, although only by 1 to 3 d (Table 2). A lower postgrazing biomass and longer grazing intervals stimulate greater pasture growth by maintaining the sward closer to the optimum state, on average (Parsons et al., 1988). Accordingly, pasture eaten was higher by 3% and silage production was higher by 211 kg of DM/ha (14%; Table 2). Intensive silage production and mowing were necessary given the low stocking rate in this treatment. Silage feeding was also 150 kg of DM/ha (29%) higher in IDEA compared with trial data, as this supplement is used to extend grazing intervals and hence promote pasture accumulation. The IDEA model predicted OM digestibility and energy content exactly, whereas lactation length and milk production were estimated within a 1% margin of error.

Table 2. Key farm characteristics recorded for a farmlet stocked at 2.2 cows/ha in Macdonald et al. (2008; MD08) and the equivalent values predicted in the integrated dairy enterprise analysis (IDEA) model and a linear programming (LP) model of the same farming system

	Source of output				
Variable	MD08	IDEA	Diff.[1] (%)	LP	Diff.[1] (%)
Pregrazing mass (kg of DM/ha)	3,300	3,262	−1	3,017	−9
Postgrazing mass (kg of DM/ha)	2,265	2,103	−8	2,038	−11
Grazing interval (d)					
Winter	58	60	+3	74	+21
Spring	31	32	+3	30	−3
Summer	23	26	+12	26	+12
Autumn	26	29	+10	26	0
Pasture eaten (kg of DM/ha)	12,098	12,493	+3	12,728	+5
Silage conserved (kg of DM/ha)	1,257	1,468	+14	421	−199
Silage eaten (kg of DM/ha)	354	504	+29	421	+16
Farm topped (%)	90	93	+3	80	−13
OM digestibility (% of DM)	76	76	0	76	0
Pasture ME (MJ/kg of DM)	11	11	0	11	0
Lactation length (d)	291	287	−1	309	+6
Annual milk production (kg/cow)	5,032	5,029	−1	5,269	+4
Annual milk production (kg/ha)	11,071	11,063	−1	11,592	+4

[1] Percentage difference between the value of the farm characteristic in Macdonald et al. (2008) and the model prediction.

The LP results for the low stocking rate treatment demonstrate that its inherent rigidity reduces its ability to accurately predict meaningful outcomes when moving away from calibrated scenarios. Postgrazing pasture mass was lower (11%) than reported in the trial data, as residuals were not determined during optimization of grazing management. Grazing intervals were 12% and 21% higher in summer and winter, respectively. Pasture eaten was 5% higher, but silage conservation was around 200% lower. Moreover, with the low stocking rate treatment, lactation length was 18 d (6%) longer in the LP, as the presence of substantial inflexibility in the feed component of the LP (e.g., fixed pasture growth and digestibility) meant that cow management was altered to a greater extent, relative to IDEA, to maximize production.

Primary reasons for large deviations with silage management in the LP model are discussed above. However, an additional driver here was that silage production confers no benefit for subsequent pasture growth or quality in the LP model. Accordingly, all silage conserved was eaten in each scenario for the LP model (Tables 1 and 2). In contrast, optimization of pasture residual mass in IDEA allowed defoliation through silage production to confer benefits for subsequent pasture growth and the digestibility of this biomass. Thus, the amount of silage conserved was greater than the amount of silage eaten in the IDEA solutions for the 2.2 cows/ha (Table 2) and 3.1 cows/ha (Table 1) systems, as identified by MD08. The same effect reduced the incentive to mow pasture in the LP model to improve pasture quality within the 2.2 cows/ha treatment (Table 2).

Comparison of IDEA and LP with Stocking Rate Treatment of 4.3 cows/ha in MD08

The IDEA model provided a good prediction of trial data for the high stocking rate treatment from MD08, as well. Pre- and postgrazing biomass levels were lower by 4 and 5%, respectively. These negative deviations reflect that the maintenance of lower postgrazing residual levels under this treatment promoted pasture growth due to reduced shading and hence less impairment of photosynthetic efficiency. This effect was present in the MD08 trial data and captured in IDEA by incorporating output from the detailed pasture simulation model. Pasture eaten was predicted with good precision, but silage production was overestimated by 66%, although the deviation was low in absolute terms (41 kg of DM/ha; Table 3). The incorporation of output from the detailed pasture simulation model in IDEA also allowed the relationship between lower postgrazing residuals and improved OM digestibility observed in the high stocking rate treatment in MD08 to be captured (Table 3). Moreover, cow attributes were predicted accurately, with lactation length and milk production within 1% of trial data.

In contrast, the LP model was infeasible with a stocking rate of 4.3 cows/ha. This reinforces how the rigid structure of a less-detailed LP model of a dairy farm

Table 3. Key farm characteristics recorded for a farmlet stocked at 4.3 cows/ha in Macdonald et al. (2008; MD08) and the equivalent values predicted in the integrated dairy enterprise analysis (IDEA) model (a linear programming model of the same farming system was infeasible in this scenario)

	Source of output		
Variable	MD08	IDEA	Diff.[1] (%)
Pregrazing mass (kg of DM/ha)	3,530	3,402	−4
Postgrazing mass (kg of DM/ha)	1,767	1,677	−5
Grazing interval (d)			
Winter	77	75	−3
Spring	32	37	+13
Summer	30	32	+6
Autumn	35	38	+8
Pasture eaten (kg of DM/ha)	16,597	16,633	+1
Silage conserved (kg of DM/ha)	65	106	+66
Silage eaten (kg of DM/ha)	876	809	−8
Purchased silage (kg of DM/ha)	812	703	−13
Farm topped (%)	0	0	0
OM digestibility (% of DM)	79	78	−1
Pasture ME (MJ/kg of DM)	11.4	11.4	0
Lactation length (d)	221	224	+1
Annual milk production (kg/cow)	3,448	3,462	+1
Annual milk production (kg/ha)	14,828	14,889	+1

[1] Percentage difference between the value of the farm characteristic in Macdonald et al. (2008) and the model prediction.

restricts its ability to flexibly represent alternative farm plans, particularly relative to IDEA.

Use of IDEA to Generate Cost-Effective Responses to Constraints on GHG Load

New Zealand dairy farms account for around 17% of the country's GHG-e (MfE, 2010). A focus toward reducing GHG-e on these farms is consistent with the Climate Change Response (Moderated Emission Trading) Amendment Act 2009. The IDEA model was used to identify how farm management should change in response to simulated reductions in baseline GHG-e of 10, 20, and 30%. This range of reductions was simulated given potential heterogeneity in the amount of mitigation required across farms and to demonstrate the flexibility of the model.

The characteristics of a representative Waikato farm were drawn from information for Waikato farms in the DairyBase database for the 2009–2010 milking season. Prices and costs for this milking season were also used. This farm has medium-input intensity, importing between 10 and 20% of feed. The IDEA model was modified to incorporate GHG-e through the addition of Intergovernmental Panel on Climate Change (IPCC) methodology calculations (IPCC, 2006) and emission factors specific for New Zealand obtained from MfE (2008).

Two types of mitigations are incorporated in the simulations to sharpen the focus on appropriate management response. A type 1 mitigation (**T1M**) involves de-intensification; T1M in the model are reductions in stocking rate, supplement, or nitrogen fertilizer application. A type 2 mitigation (**T2M**) is an abatement strategy that does not explicitly require de-intensification; T2M in the model are the use of improved reproduction, turnip (*Brassica rapa*) crops, nitrification inhibitors, and a restricted grazing regimen incorporating a feed pad or a stand-off pad (Eckard et al., 2010).

Table 4 presents key farm system outputs computed in IDEA for the simulated reductions in GHG-e. The base scenario reports the model solution calibrated to survey data, with no T2M available; T2M were available in all other scenarios. The IDEA output indicated that de-intensification was a key strategy to reduce GHG-e (Table 4). Stocking rate decreased by 9, 17, and 26% at simulated reductions of 10, 20, and 30%, respectively (Table 4). Also, application of N fertilizer decreased by 17, 54, and 97% at simulated reductions of 10, 20, and 30%, respectively (Table 4). Moreover, the level of purchased supplement (PKE) decreased as more abatement was required, because reductions in stocking rate decreased feed demand (Table 4). The availability of T2M increased farm profit by $13/ha in the baseline solution (Table 4). This involved the use of improved reproductive management, which reduced the cost of managing more young stock by decreasing the replacement rate (Table 4), but added costs associated with improved heat detection and increased body condition at calving. Improved reproduction was the only T2M used in any simulated scenario.

Table 4. Key farm system output from the integrated dairy enterprise analysis (IDEA) model for simulated reductions in greenhouse gas (GHG) emissions of 0 (base), 10, 20, and 30% for a representative Waikato dairy farm

	Baseline scenario[1]		Reduction in GHG emissions (%)		
Variable	Base	Base + T2M	10	20	30
Farm profit ($/ha)	1,257	1,270	1,209	1,077	903
Farm profit [$/kg of milk solids (MS)]	1.21	1.22	1.25	1.22	1.14
Stocking rate (cows/ha)	3	3	2.73	2.48	2.23
Milk production (kg of MS/cow)	346	347	354	356	356
Milk production (kg of MS/ha)	1,038	1,041	966	881	794
Lactation length (d)	278	277	288	290	290
Grazed pasture eaten (t of DM/ha)	12.58	12.61	11.82	10.75	9.59
Grass silage eaten (t of DM/ha)	0.64	0.61	0.32	0.17	0.26
Maize silage eaten (t of DM/ha)	0.37	0.37	0.37	0.37	0.37
Palm kernel expeller eaten (t of DM/ha)	1.54	1.54	1.41	1.27	1.16
N fertilizer applied (kg of N/ha)	112	112	93	52	3
Crop area (% of area)	2.5	2.5	2.5	2.5	2.5
Replacement rate (%)	23.3	18.3	17.8	17.5	17.5
Methane emissions (kg of CH_4/ha)	295.08	295.69	274.04	247.64	220.8
N_2O emissions (kg of N_2O/ha)	8.68	8.64	7.79	6.49	5.09
GHG emissions (kg of CO_2-eq/ha)	12,745	12,546	11,471	10,196	8,922
Mitigation practices used	—	Improved reproduction	Improved reproduction	Improved reproduction	Improved reproduction

[1] Base = baseline level of production with type 2 mitigations (T2M; abatement strategies) unavailable; Base + T2M = baseline level of production with T2M available.

CONCLUSIONS

We described a detailed nonlinear optimization model of a New Zealand dairy farming system. This framework is notable for its inclusion of pasture residual mass, intake regulation, and pasture utilization as key decision variables. Validation of the model showed that the detailed representation of key biophysical relationships in IDEA had 2 key benefits compared with the less-detailed LP model. First, the model has an enhanced capacity to provide reasonable predictions outside of calibrated scenarios. Second, the flexibility of management plans in IDEA helps avoid the infeasibility of the model when significant perturbations to calibrated scenarios are simulated. Interesting extensions of the model include the incorporation of tactical decision making (Pannell, 1996) and the representation of dairy farms in different regions of New Zealand. However, the IDEA framework could benefit from further biophysical research. Key issues are the relationship between pasture growth and quality and different postgrazing residuals within New Zealand's key dairy regions and the association between time off pasture and cow intake.

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